

PREPARED FOR SUBMISSION TO JHEP

Hydrodynamics in 1+1 dimensions with gravitational anomalies

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ABSTRACT: The constraints imposed on hydrodynamics by the structure of gauge and gravitational anomalies are studied in two dimensions. By explicit integration of the consistent gravitational anomaly, we derive the equilibrium partition function at second derivative order. This partition function is then used to compute the parity-violating part of the covariant energy-momentum tensor and the transport coefficients.

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1 Introduction

The recent studies of the role played by gauge anomalies in relativistic hydrodynamics have revealed unexpected modifications in the constitutive relations of the anomalous currents [1, 2]. These give rise to parity violating macroscopic phenomena such as the chiral magnetic and chiral vortical effects [3, 4]. It has also been pointed out in [5] that the temperature corrections to the constitutive relations is related to the mixed gravitational anomaly. The guiding principle which usually has served to derive the form of constitutive relations is the principle of entropy increase. However, as some of the new parity odd transport coefficients have even signature under time reversal, they produce non-dissipative effects and do not contribute to entropy increase. Arguments based on the requirement of adiabaticity have just provided a better understanding of anomaly induced effects [6, 7], and they have helped as well to make progress in the classification of the corrections in the derivative expansion to second order fluids dynamics [8].

Very recently, it has been argued that it is possible to find all the corrections to the constitutive relations without dissipative coefficients from the knowledge of the equilibrium thermodynamics [9, 10]. This has been checked in various examples [9–12] where the form of the partition function is determined by general arguments of gauge and diffeomorphism invariance.

In this note we consider the inclusion of the effects of the gravitational anomaly on the hydrodynamics in 1+1 dimensions, about which little is known. Previous studies of the effects of pure gravitational anomalies have focused on the connection of these effects with viscoelastic and thermal transport phenomena in topological insulators [13, 14]. Here, our interest is rather in the implications of gravitational anomalies in a relativistic setting, where a theory of chiral fermions is put in an external gravitational field. Specifically, we will construct an anomalous partition function in a curved manifold, and by following the methods of [9, 10] derive the anomaly induced constitutive relations. Our main finding is a novel term of second order in the derivatives of the velocity fluid which, in the Landau frame, shows up in the constitutive relation for current density.

The approach that we have followed for the computation of the partition function is based on the integration of the formula for the consistent gravitational anomaly in the special case of a time-independent background. As shown in section 2, in such a case the integration yields a local functional depending on the gravitational sources, but since the consistent anomaly is not generally covariant, the quantities obtained by functional differentiation with respect to the external sources do not transform like a tensor. As it is well known, the addition of a specific term known as Bardeen’s polynomial [15] produces an energy-momentum tensor that is generally covariant, although no longer derivable from a partition function.

With the partition function at hand, we apply the systematic method presented in [9, 10] for constraining the constitutive relations of hydrodynamics. It is worth mentioning that the necessity to shift from consistent to covariant currents is not peculiar to the gravitational case, as it already shows up in [9–11], where only the role of gauge anomalies is considered¹. Finally, in section 3 the covariant energy-momentum tensor is used to derive the modifications of the constitutive relations which follow from the gravitational anomaly.

2 The gravitational anomaly and its contribution to the partition function

In two dimensions the most general static metric and gauge field which are preserved by the Killing vector ∂_t may be written as

$$\begin{aligned} ds^2 &= -e^{2\sigma(x)} (dt + a_1(x)dx)^2 + g_{11}dx^2, \\ \mathcal{A} &= \mathcal{A}_0(x)dt + \mathcal{A}_1(x)dx. \end{aligned} \tag{2.1}$$

We are interested in discussing the role of the gravitational anomaly in the hydrodynamics of a chiral charged fluid. The anomalous conservation law for the consistent energy-momentum tensor is [15, 17]

$$\nabla_\mu T^{\mu\nu} = D \epsilon^{\gamma\delta} \partial_\alpha \partial_\gamma \Gamma_{\delta\rho}^\alpha g^{\rho\nu}, \tag{2.2}$$

where $\epsilon^{01} = \frac{1}{\sqrt{-g}} = (e^\sigma \sqrt{g_{11}})^{-1}$, and the coefficient D is $\pm 1/(96\pi)$ for a Weyl fermion. The fact that this tensor is constructed from the variation of a functional action W constrains the form of the anomalous divergence through a consistency condition which follows from the commutation relations between infinitesimal coordinate transformations [15]. Such a transformation is specified by infinitesimal parameters $\xi^\mu(x)$, and may be decomposed into two parts when acting on non-tensorial quantities, $\delta_\xi = \mathcal{L}_\xi + \delta_\Lambda$. For instance, by using matrix notation for the vielbein and the connection, $(E)_\mu^a \equiv e_\mu^a$, $(\Gamma_\lambda)_\mu^\nu \equiv \Gamma_{\lambda\mu}^\nu$, these transform as [15]

$$\begin{aligned} \delta_\xi E &= \mathcal{L}_\xi E - \Lambda E, \\ \delta_\xi \Gamma_\lambda &= \mathcal{L}_\xi \Gamma_\lambda + [\Gamma_\lambda, \Lambda], \end{aligned} \tag{2.3}$$

¹For a careful discussion of the distinction between the role played of consistent and covariant currents, see also [16].

where \mathcal{L}_ξ is the Lie derivative and Λ is the matrix $(\Lambda)_\mu^\nu \equiv -\partial_\mu \xi^\nu$. This Λ may be viewed as the matrix specifying a gauge transformation on the connection. It is possible to express the effective action W by introducing a set of fields H which transform non-linearly under the gauge part δ_Λ for the connection in (2.3). These fields H are given in terms of the vielbein by the matrix relation $e^H = E$. Therefore, the effective anomalous action is given by [15, 18]

$$W_{\text{anom}}[H, \Gamma] = D \int d^2x \sqrt{-g} \int_0^1 ds \text{Tr} \left(H \partial_\rho \Gamma_\lambda(s) \epsilon^{\rho\lambda} \right), \quad (2.4)$$

where

$$\Gamma_\lambda(s) = e^{-sH} \Gamma_\lambda e^{sH} + e^{-sH} \partial_\lambda e^{sH}. \quad (2.5)$$

For the background metric (2.1) the integration over s may be explicitly performed. The result is expressed, remarkably, as a local functional of the metric fields $\sigma(x)$, $a_1(x)$, $g_{11}(x)$ and their derivatives up to second order

$$W_{\text{anom}}^{(2)} = \int dt \int dx P_{\text{anom}}, \quad (2.6)$$

where the integrand turns out to be

$$\begin{aligned} \frac{1}{D} \frac{1}{\sqrt{-g}} P_{\text{anom}} &= \frac{e^{2\sigma} a_1 \sigma''}{2g_{11} (e^\sigma - \sqrt{g_{11}})} (\ln g_{11} - 2\sigma) \\ &+ \frac{e^{2\sigma} a_1 (g'_{11} \sigma' - 2g_{11} \sigma'^2)}{4g_{11}^2 (e^\sigma - \sqrt{g_{11}})^2} \\ &\times [e^\sigma (2 - \ln g_{11} + 2\sigma) + 2\sqrt{g_{11}} (-1 + \ln g_{11} - 2\sigma)] \\ &+ \frac{e^{2\sigma} a_1' \sigma'}{2g_{11}^{3/2} (e^\sigma - \sqrt{g_{11}})} [2e^\sigma + \sqrt{g_{11}} (-2 + \ln g_{11} - 2\sigma)]. \end{aligned} \quad (2.7)$$

This locality of the anomalous effective action has also been found in [6, 7, 10, 11], where consistent static currents were derived from a local contribution to the partition function. From a diagrammatic point of view, the origin of the locality in the static case can be traced to the fact that restricting the amplitudes to zero frequency turns the rational dependence on the external momenta into a polynomial one.

Since at thermal equilibrium with inverse temperature β_0 the euclidean vacuum functional W is related with the thermodynamic potential Ω through $W \equiv -\beta_0 \Omega$, it follows that the function P_{anom} represents an anomalous contribution to the pressure. Following the notation in [10], we write the anomalous contribution to the partition function as

$$W_{\text{anom}}^{(2)} = \frac{1}{T_0} \int dx P_{\text{anom}} = -\beta_0 \Omega[g_{\mu\nu}]. \quad (2.8)$$

The components of the consistent energy-momentum tensor are easily obtained by functional differentiation

$$T^{\mu\nu} = \frac{2T_0}{\sqrt{-g}} \frac{\delta W}{\delta g_{\mu\nu}(x)} = -\frac{2}{\sqrt{-g}} \frac{\delta \Omega}{\delta g_{\mu\nu}(x)}, \quad (2.9)$$

yielding the following simple expressions

$$T_{00} = -\frac{T_0 e^{2\sigma}}{\sqrt{-g}} \frac{\delta W}{\delta \sigma} = D \frac{e^{3\sigma}}{g_{11}^{5/2}} (-a'_1 g'_{11} + g_{11} a''_1), \quad (2.10)$$

$$T_0^1 = \frac{T_0}{\sqrt{-g}} \frac{\delta W}{\delta a_1} = D \frac{e^\sigma}{g_{11}^{5/2}} (g'_{11} \sigma' - 2g_{11} \sigma'^2 - g_{11} \sigma''), \quad (2.11)$$

$$T^{11} = \frac{2T_0}{\sqrt{-g}} \frac{\delta W}{\delta g_{11}} = -D \frac{2e^\sigma}{g_{11}^{5/2}} a'_1 \sigma'. \quad (2.12)$$

We have computed the components that, according to [10], should be invariant under Kaluza-Klein gauge transformations. These transformations are redefinitions of time, $t \rightarrow t' = t + \phi(x)$, without change in the spatial coordinate, that preserve the form of the metric if a_1 transforms as $\delta a_1 = -\phi'$. As discussed in [10], the invariant combinations of the components of a tensorial quantity $V^{\mu\nu}$ correspond to the purely spatial indices V^{11} , the lower temporal indices V_{00} , and V_0^1 . Clearly, the above components T_{00} and T^{11} do not satisfy this property because of their specific dependence on the derivatives of a_1 .

This can be traced to the fact that, under a spatial diffeomorphism, the consistent components that we have computed do not transform as tensorial quantities. Fortunately, there is a covariant modification $\tilde{T}^{\mu\nu}$ which may be obtained through a shift $Y^{\mu\nu}$ of the original combination [15],

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + Y^{\mu\nu}. \quad (2.13)$$

This is called the covariant energy-momentum tensor and its divergence will also be covariant. It is given by [15, 17]

$$\nabla_\mu \tilde{T}^{\mu\nu} = -D \epsilon^{\nu\rho} \partial_\rho R, \quad (2.14)$$

where R is the scalar curvature,

$$R = \frac{1}{g_{11}^2} (g'_{11} \sigma' - 2g_{11} \sigma'^2 - 2g_{11} \sigma''). \quad (2.15)$$

It is possible to show that, in general, $Y^{\mu\nu}$ must be a symmetric local quantity depending on second derivatives of the metric [15]. For our background metric we find

$$\frac{1}{D} Y^{00} = \frac{e^{-\sigma}}{g_{11}^{5/2}} [a'_1 (g'_{11} + 2e^{2\sigma} a_1^2 \sigma') - g_{11} (a''_{11} - 4a_1 \sigma'^2 + 2a_1 \sigma'')], \quad (2.16)$$

$$\frac{1}{D} Y^{01} = \frac{e^{-\sigma}}{g_{11}^{5/2}} [-2e^{2\sigma} a_1 a'_1 \sigma' - 2g_{11} \sigma'^2 + g_{11} \sigma''], \quad (2.17)$$

$$\frac{1}{D} Y^{11} = \frac{2e^\sigma}{g_{11}^{5/2}} a'_1 \sigma'. \quad (2.18)$$

With these results at hand, we finally obtain the anomalous covariant energy-momentum tensor. Their components are given by

$$\frac{1}{D}\tilde{T}^{00} = \frac{2e^{-\sigma}a_1}{g_{11}^{5/2}}(g'_{11}\sigma' - 2g_{11}\sigma''), \quad \tilde{T}_{00} = 0, \quad (2.19)$$

$$\frac{1}{D}\tilde{T}^{01} = -\frac{e^{-\sigma}}{g_{11}^{5/2}}(g'_{11}\sigma' - 2g_{11}\sigma''), \quad \frac{1}{D}\tilde{T}_0^1 = \frac{e^\sigma}{g_{11}^{5/2}}(g'_{11}\sigma' - 2g_{11}\sigma''), \quad (2.20)$$

$$\tilde{T}^{11} = 0. \quad (2.21)$$

The gauge invariance of $\tilde{T}^{\mu\nu}$ with respect to Kaluza-Klein gauge transformations, absent in (2.10) and (2.12), is now manifest. It is also worthwhile checking the tensorial character of $\tilde{T}^{\mu\nu}$. Under a diffeomorphism generated by $\xi^\mu = (0, \xi(x))$ the Lie derivative of the metric with respect to ξ produces the changes

$$\delta\sigma(x) = \xi(x)\sigma'(x), \quad (2.22)$$

$$\delta a_1(x) = \xi(x)a'_1(x) + \xi'(x)a_1(x), \quad (2.23)$$

$$\delta g_{11}(x) = \xi(x)g'_{11} + 2\xi'(x)g_{11}, \quad (2.24)$$

which may be used to compute the induced change $\delta\tilde{T}^{\mu\nu}(x)$. The calculation of these components using the above rules shows that $\delta\tilde{T}^{\mu\nu}(x)$ precisely coincides with the expression for $\mathcal{L}_\xi\tilde{T}^{\mu\nu}$. This provides a non-trivial check of the whole computation because, in principle, $\delta\tilde{T}^{\mu\nu}(x)$ receives contributions proportional to $\xi''(x)$ which cancel out in the final result.

In order to complete the description of an anomalous charged fluid, we must also include the effects of the gauge anomaly, which already show up at zero derivative order. The consequences of the gauge anomaly on the hydrodynamics in 1+1 dimensions have been studied in [11, 19]. The most general partition function which encodes the effect of the gauge anomaly at zero order in the derivative expansion takes the form [11]

$$W^{(0)} = W_{\text{anom}}^{(0)} + W_{\text{inv}}^{(0)} = -\frac{C}{T_0} \int \mathcal{A}_0 (\mathcal{A}_1 - \mathcal{A}_0 a_1) dx - C_2 T_0 \int a_1 dx, \quad (2.25)$$

where C is the coefficient in the consistent anomaly, $\nabla_\mu J^\mu = C\epsilon^{\mu\nu}\partial_\mu\mathcal{A}_\nu$, and C_2 is an arbitrary coefficient. Hence $W_{\text{anom}}^{(0)} + W_{\text{anom}}^{(2)}$ encompasses the effects of the anomalies in the background (2.1). This action produces the consistent current

$$J^\mu = \frac{T_0}{\sqrt{-g}} \frac{\delta W}{\delta \mathcal{A}_\mu}, \quad (2.26)$$

that, when shifted by $C\epsilon^{\mu\lambda}\mathcal{A}_\lambda$, yields the gauge covariant current

$$\tilde{J}^\mu = J^\mu + C\epsilon^{\mu\lambda}\mathcal{A}_\lambda, \quad \nabla_\mu \tilde{J}^\mu = C\epsilon^{\mu\nu}\mathcal{F}_{\mu\nu}. \quad (2.27)$$

Its components are given by [11]

$$\tilde{J}_0 = 0, \quad \tilde{J}^1 = -2C \frac{e^{-\sigma}}{\sqrt{g_{11}}} \mathcal{A}_0. \quad (2.28)$$

One immediate consequence of the addition of $W^{(0)}$ to $W^{(2)}$ is that now the non-conservation law for the consistent energy-momentum “tensor” adopts the form

$$\nabla_\mu T^{\mu\nu} = D \epsilon^{\gamma\delta} \partial_\alpha \partial_\gamma \Gamma_{\delta\rho}^\alpha g^{\rho\nu} + \mathcal{F}^{\nu\alpha} \tilde{J}_\alpha, \quad (2.29)$$

as follows from the behavior of the total functional $W = W^{(0)} + W^{(2)}$ under diffeomorphisms. Thus the conservation law for the covariant counterpart reads

$$\nabla_\mu \tilde{T}^{\mu\nu} = -D \epsilon^{\nu\rho} \partial_\rho R + \mathcal{F}^{\nu\alpha} \tilde{J}_\alpha. \quad (2.30)$$

By adding to (2.20) the zero order contribution which follows from (2.25), the parity odd part of the stress tensor reads

$$\tilde{T}_0^1 = \frac{e^{-\sigma}}{\sqrt{g_{11}}} (C \mathcal{A}_0^2 - T_0^2 C_2) + D \frac{e^\sigma}{g_{11}^{5/2}} (g'_{11} \sigma' - 2g_{11} \sigma''), \quad (2.31)$$

while the remaining Kaluza-Klein gauge invariant components \tilde{T}_{00} and \tilde{T}^{11} vanish.

3 Anomaly induced transport coefficients in the Landau frame

We now seek the form of the constitutive relations that arise from (2.28) and (2.31). At zero order in the derivative expansion, the time-independent equilibrium fluid fields in the background (2.1) are given by

$$\begin{aligned} u_K^\mu &= e^{-\sigma} (1, 0), \\ T &= T_0 e^{-\sigma}, \\ \mu &= \mathcal{A}_0 e^{-\sigma}, \end{aligned} \quad (3.1)$$

where μ denotes the chemical potential. Generally, as explained in [10] and [9], these quantities receive derivative corrections which can be expressed in terms of some specific combinations of covariant derivatives of the background data. Such combinations are non zero at equilibrium and can be determined from the knowledge of the partition function. The derivative terms of the constitutive relations that arise in this way are static susceptibilities rather than transport coefficients related to genuine irreversible processes.

In 1+1 dimensions the velocity field may also receive corrections of zero order in the derivative expansion [11]. This is due to the existence of a non-zero independent vector field

$$\tilde{u}_K^\mu \equiv \epsilon^{\mu\nu} u_{K\nu} = \left(-\frac{a_1}{\sqrt{g_{11}}}, \frac{1}{\sqrt{g_{11}}} \right), \quad \tilde{u}_{K0} = 0, \quad \tilde{u}_K^1 = \frac{1}{\sqrt{g_{11}}}, \quad (3.2)$$

which may be constructed by Hodge duality. On the other hand, the second order derivative contribution in (2.31) contains a quantity that may be written as the Laplacian on σ with respect to the one-dimensional metric g_{11}

$$\nabla^2 \sigma = \frac{1}{\sqrt{g_{11}}} \frac{d}{dx} \left(\sqrt{g_{11}} \frac{1}{g_{11}} \frac{d\sigma}{dx} \right) = \frac{1}{2g_{11}^2} (-g'_{11} \sigma' + 2g_{11} \sigma''). \quad (3.3)$$

If one considers an arbitrary normalized fluid velocity, this term is precisely the second order scalar $(u^\mu \nabla^\rho - u^\rho \nabla^\mu) \nabla_\mu u_\rho$ for $u^\mu = u_K^\mu$

$$(u_K^\mu \nabla^\rho - u_K^\rho \nabla^\mu) \nabla_\mu u_{K\rho} = \nabla^2 \sigma. \quad (3.4)$$

Therefore, using (3.1) and (3.2), we can express the anomalous contributions (2.28) and (2.31) in terms of the fluid fields and their derivatives evaluated at equilibrium

$$\begin{aligned} J_{\text{anom}}^\mu &= -2C\mu \tilde{u}^\mu, \\ T_{\text{anom}}^{\mu\nu} &= [C_2 T^2 - C\mu^2 + 2D(u^\mu \nabla^\rho - u^\rho \nabla^\mu) \nabla_\mu u_\rho] (u^\mu \tilde{u}^\nu + u^\nu \tilde{u}^\mu). \end{aligned} \quad (3.5)$$

These expressions are precisely the parity odd corrections to the constitutive relations. Note that they are expressed in the frame defined by the Killing direction of the metric (2.1) used to compute the anomalous thermodynamics. Together with the even contribution from the zero derivative partition function, they imply the following forms for the current and the energy-momentum tensor

$$\begin{aligned} J^\mu &= nu^\mu + J_{\text{anom}}^\mu, \\ T^{\mu\nu} &= \varepsilon u^\mu u^\nu + pP^{\mu\nu} + T_{\text{anom}}^{\mu\nu}, \end{aligned} \quad (3.6)$$

where $P^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$ and p, ε, n are, respectively, the pressure, energy and charge densities which follow from the zero order partition function.

In contrast, in 1+1 dimensions the tensor corrections to the perfect fluid part of the energy-momentum tensor necessarily vanish in the Landau frame², so that all the modifications to the constitutive relations appear in the current J^μ , and perhaps also in the pressure

$$\begin{aligned} J^\mu &= nu^\mu + \chi \tilde{u}^\mu, \\ T^{\mu\nu} &= \varepsilon u^\mu u^\nu + (p + \Delta p)P^{\mu\nu}. \end{aligned} \quad (3.7)$$

The transformation to this frame may be written as

$$u^\mu = u_K^\mu \cosh \gamma + \tilde{u}_K^\mu \sinh \gamma, \quad (3.8)$$

while the temperature and chemical potential will not receive corrections at linear order in the anomaly coefficients C, D and C_2 . Plugging this equation into (3.7) and comparing with (3.6) the parameter γ may be determined. One finds

$$\gamma = \frac{1}{\varepsilon + p} [C_2 T^2 - C\mu^2 + 2D(u^\mu \nabla^\rho - u^\rho \nabla^\mu) \nabla_\mu u_\rho] + \text{second order corrections}. \quad (3.9)$$

This gives the anomaly induced coefficients of the constitutive relations in the Landau frame

$$\begin{aligned} \chi &= -C_2 \frac{nT^2}{\varepsilon + p} + C \left(\frac{n\mu^2}{\varepsilon + p} - 2\mu \right) - 2D \frac{n(u^\mu \nabla^\rho - u^\rho \nabla^\mu) \nabla_\mu u_\rho}{\varepsilon + p}, \\ \Delta p &= 0. \end{aligned} \quad (3.10)$$

Note that there are also parity even contributions at two-derivative order which we have not computed. Without knowledge of the specific form of the second order partition function these corrections are undetermined.

²The three constraints $\Delta T^{\mu\nu} u_\nu = 0$, $g_{\mu\nu} \Delta T^{\mu\nu} = 0$, define a vanishing $\Delta T^{\mu\nu}$.

4 Conclusion

In this note we have computed, in 1+1 dimensions, the contribution of the gravitational anomaly to the partition function in the most general background with external sources consistent with thermodynamic equilibrium. The energy-momentum tensor at equilibrium has been evaluated as a variation of the partition function and then corrected by the appropriate term to make it generally covariant. By following the methods of [10] and [9], we have obtained the corresponding contribution to the constitutive relations without resort to the principle of entropy increase. The coefficient of the temperature correction does not seem to be determined by the anomaly coefficients, and it appears as an independent quantity.

It would be interesting to perform a similar treatment in order to study the effects of the mixed gauge-gravitational anomalies in 3+1 dimensions up to third order in the derivative expansion.

Acknowledgements

I would like to thank Juan L. Mañes for many helpful discussions. This work was supported by funds from the Spanish Ministry of Science and Technology under grant FPA2009-10612, the Consolider-Ingenio Programme CPAN (CSD2007-00042), and the Basque Government under grant IT559-10.

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